Impedance Modeling of a Whisker Mounted in a Rectangular Waveguide

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Abstract—**Theoretical impedance modeling of metal whisker, which is extensively used in millimeter- and submillimeter-wave Schottky diode mixers and frequency multiplier circuits, is presented. The moment method with Galerkin's technique is employed to analyze this structure with the help of the dyadic Green's function in a rectangular waveguide. Effects of whiskers with various types of shape on the embedding impedance are investigated. Theoretical results show good agreement with related results published before.**

I. Introduction

BOTH a metal whisker and a straight post in a waveguide have found extensive applications in constructing millimeter wave circuits. A series of publications on analysis of the one-directional uniform post have appeared during the past two decades, but few publications on modeling of the whisker have yet appeared. Some experimental studies on the mount have been done, see for example [1], [2]. This letter reports our three-dimensional modeling of a metal whisker that is mounted in a rectangular waveguide. The moment method with the dyadic Green's function is employed to model the radiation feature of the whisker with arbitrary geometry in the waveguide. The waveguide walls and the whisker are assumed to be perfect conductors and main attention is given to the electrical property of the small whisker. The whisker geometry is approximated by a group of arc wires and straight wires. The analytical results show good agreement with related results published in the literature and can be expected to provide fundamental guidance for the circuit design utilizing a whisker mount.

II. Formulation

The whisker configuration under consideration is shown in Fig. 1. Applying a voltage V across the infinitesimal excitation gap, the excited current $\boldsymbol{J}(\boldsymbol{r})$ along the whisker surface together with the impressed field $E^{\mathrm{im}}(\boldsymbol{r})$ should satisfy the following integral equation

$$E^{\mathrm{im}}(\boldsymbol{r}) = \boldsymbol{j}\omega\mu \int_{\text{whisker}} \boldsymbol{G}(\boldsymbol{r} \mid \boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}')\, dv', \qquad (1)$$

where $\boldsymbol{G}(\boldsymbol{r}|\boldsymbol{r}')$ is the electric-type dyadic Green's function in rectangular waveguide. Several techniques are available to solve the above integral equation, but the method of moments

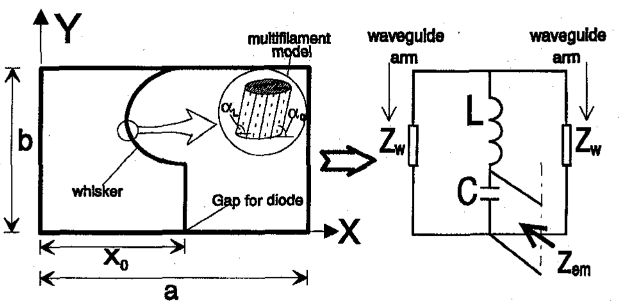

Fig. 1. A whisker mount with multifilament model and lumped equivalent circuit for the case that only TE_{10} mode propagates in the waveguide. ———: current filaments, - - - -: field observation points.

is employed here because of its versatility to deal with the whisker's complex geometry. Since the angular variation of the post surface current makes little contribution to the input impedance of the wire post with an electrically small diameter [3], the current distribution of the whisker is represented by a group of surface current multifilaments parallel to the whisker axis. Furthermore, each filament is approximated by a series of straight lines.

Without loss of generality of the analysis, only formulations for the case of transverse current distribution is presented below. Defining both the basis function $\boldsymbol{B}(\boldsymbol{r})$ and weighting function $\boldsymbol{W}(\boldsymbol{r})$ as

$$\boldsymbol{B}(r) = \boldsymbol{W}(r) = \hat{u}_n P^n(r), \qquad n = 1, \cdots, L, \qquad (2)$$

where

$$P^n(r) = \begin{cases} 1, & \text{for } l_{n-1} \le l \le l_n, \\ 0, & \text{otherwise,} \end{cases} \qquad (3)$$

and $\hat{u}_n$ denotes a unit vector along the whisker axis, L is the total number of straight segments. Applying the moment method to (1), a matrix equation can be obtained for the current distribution

$$[Z_l^q][J_l] = [V_q], \qquad q, l = 1, \cdots, L, \qquad (4)$$

where J_l is the current expansion coefficient,

$$V_q = \int_{\Delta S_q} W_q(r) E^{\mathrm{im}}(r) dl, \qquad (5)$$

$$Z_l^q = \cos\alpha_q \cos\alpha_l \int_{l_q} \int_{l_l'} W_q \cdot G_{xx} \cdot B_l dl' dl$$

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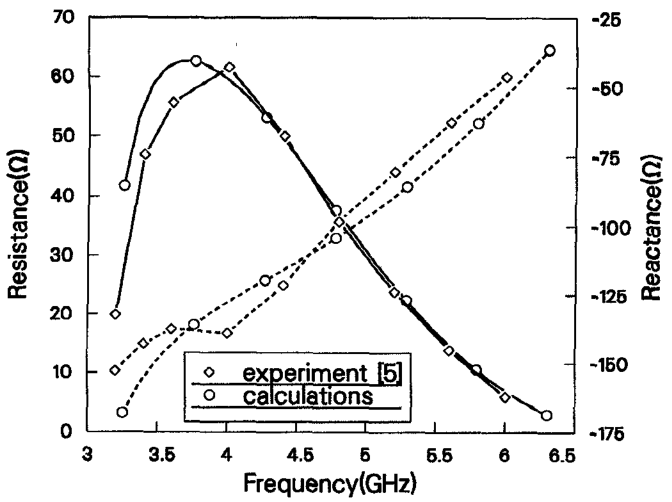

Fig. 2. Comparisons between theoretical calculations with experimental results in [5] for a straight post. Dimensions $a = 47.6$ mm, $b = 22.15$ mm, $d = 3.05$ mm, $X_0 = a/2$.

$$+\sin\alpha_q \cos\alpha_l \int_{l_q}\int_{l_l'} W_q \cdot G_{yx} \cdot B_l dl' dl$$
$$+\cos\alpha_q \sin\alpha_l \int_{l_q}\int_{l_l'} W_q \cdot G_{xy} \cdot B_l dl' dl$$
$$+\sin\alpha_q \sin\alpha_l \int_{l_q}\int_{l_l'} W_q \cdot G_{yy} \cdot B_l dl' dl, \quad (6)$$

where $G_{ij}, (i,j) = (x,y)$ are dyad components of the Green's function [4].

Upon determining of the current distribution on the whisker, the embedding impedance Z_{em} looking at the excitation point is readily evaluated [3]. In the case that the whisker is located on a cross-sectional plane of the waveguide, the structure can be described by a simplified form of equivalent circuit as shown on the right side of Fig. 1. The capacitance C and the inductance L are de-embedded from Z_{em} to be

$$C = \frac{Z_r\sqrt{\frac{2Z_i^2}{Z_r} + 2Z_r - Z_w} - \sqrt{Z_w}Z_i}{2\pi f\sqrt{Z_w}(Z_i^2 + Z_r^2)}$$
$$L = \frac{1}{(2\pi f)^2 C} - \frac{Z_w(1 + Z_i(2\pi f C))}{2Z_r(2\pi f)^2 C}, \quad (7)$$

where Z_r and Z_i are the real and imaginary part of Z_{em}, respectively, and Z_w is characteristic impedance of waveguide in terms of power-current definition.

III. Numerical Results

A computer program has been written to implement the formulations described above. The singular value decomposition (SVD) method is utilized to solve the matrix equation (4) after the generalized impedance matrix components are determined. All the infinite series, occurring in (6), are summarized by means of partial summation technique.

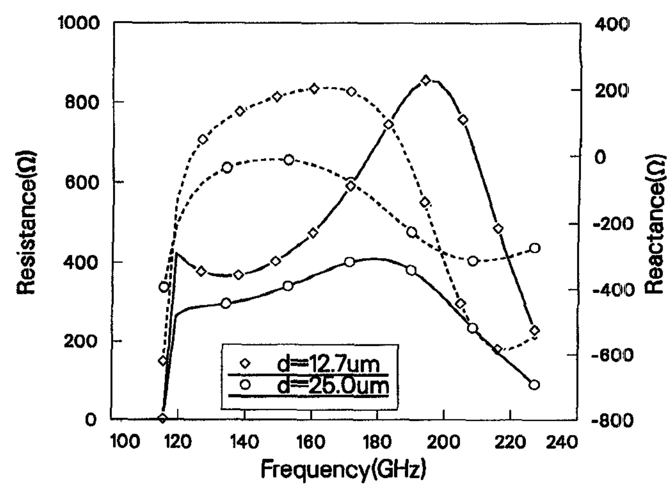

Fig. 3. Embedding impedance (Z_{em} in Fig. 1) for whiskers of different diameters. ———: resistance; – – – –: reactance. Dimensions $a = 1.3$ mm, $b = 0.33$ mm, $X_0 = a/2$, whisker length $L_w = 0.53$ mm.

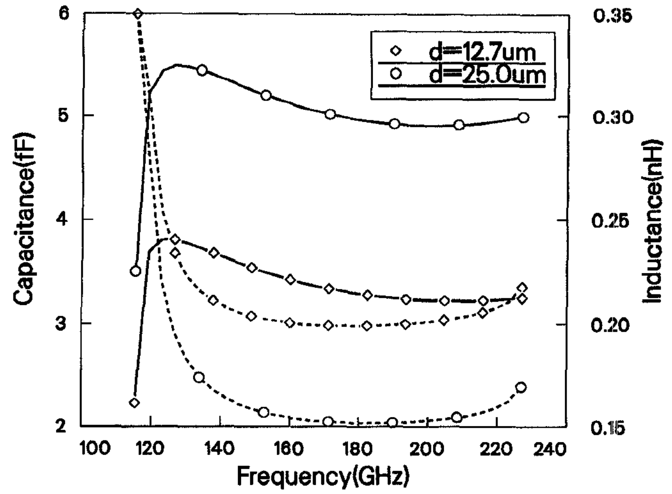

Fig. 4. Frequency dependence of the lumped-circuit equivalent parameters derived from Fig. 3 in terms of (7) and Fig. 1. ———: capacitance; – – – –: inductance. Whisker orientation: cross-sectional plane of waveguide.

There are many publications about straight wire modeling, but few results about curved wires are available in the open literature. Several structures including a straight wire and the whisker, shown in Fig. 1, have been modeled in terms of this formulation. The validity of the method is first checked by modeling straight wires. For convenience of comparison, dimensions for the straight wire mount are chosen to be coincident with those used in [5]. A good agreement between present calculations and experimental results in [5] has been obtained. Computations are then extended to cover curved-whisker modeling. Good agreement with experimental estimations in [2] has been observed. Two types of whiskers mounted in a half-height WR-4 waveguide with dimensions $a = 1.3$ mm, $b = 0.33$ mm have been modeled. The diameter (d) of one whisker is 12.7μm, $d = 25.0\mu$m for another. The variation of the embedding impedance of a whisker vs. frequency is shown in Fig. 3. Fig. 4 presents the frequency dependence of the lumped circuit equivalent parameters, L and C, as defined in Fig. 1 and (7). The effects of whisker diameter are also shown in these figures.

IV. Discussion

The analysis of a commonly used whisker mount has been outlined. It is, according to our knowledge, the first time that the theoretical modeling of a waveguide whisker mount has been carried out. The calculations are verified by the experimental measurements in [5] and experimental estimations in [2]. Results for a typical millimeter wave mount have been presented. From preceding calculations, one can see that the embedding impedance of the whisker mount varies a lot in the waveguide dominant mode frequency band even if the circuit equivalent parameters show little frequency dependence. In practical applications, the active device embedded in the mount may not be flush with the waveguide bottom. The present formulation is for this special case, but it does not limit the possibility of optimization of the device embedding impedance by a proper whisker shape.

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